Finance Study Notes - Grade 10

Simple Interest

- Interest = payment for the use of borrowed money.
- Interest is expressed as a %
- Simple interest is an amount of interest which is paid ever so often, at a fixed amout (the amount stays same).

Formula:

$$A = P(1 + in)$$

n = time periods

interest rate $\left(\frac{r\%}{100}\right)$

total amount at the end

Principal or initial amount

* Note: We can calculate any value (A, P, i, or n) if we are given the other 3 values.
* This is called appreciation (increase)

Example:

a.i) R 8600 is invested at 11% p.a (per annum) for 8 years. What is the value of the investment after those 8 years?

$A = ?$
$P = 8600$
$i = 11\% \rightarrow \frac{11}{100}$
$n = 8$

$$A = P(1 + in)$$
$$= 8600\left(1 + \frac{11}{100} \times 8\right)$$
$$= R\,16\,168$$

a.ii) How much interest was earned?

$R16168 - R8600 = R\,7568$

b) R 3600 is invested at r% p.a SI and amounts to R 4608 in 4 years. Find r.

$A = P(1+in)$

$A = R4608$
$P = R3600$
$i = r\%$?
$n = 4$

$4608 = 3600(1 + \frac{r}{100} \times 4)$

$\frac{4608}{3600} = \frac{3600(1 + \frac{4r}{100})}{3600}$

$\frac{32}{25} = 1 + \frac{4r}{100}$

$\frac{32}{25} - 1 = \frac{4r}{100}$

$100 \times \quad \frac{7}{25} = \frac{4r}{100} \times 100$

$28 = 4r$

$\therefore r = 7\%$

* You can check your answer by substituting back into the original formula.

c) An investment of R12 000 at 8% SI p.a. yielded R5760 interest in n years. Find n.

$A = P(1+in)$

$P: R12\,000$
$A: R12\,000 + R5760 = R17\,760$
$i: 8\% \rightarrow \frac{8}{100}$
$n: ?$

$17\,760 = 12\,000(1 + \frac{8}{100} \times n)$

$\frac{17\,760}{12000} = \frac{12\,000(1 + \frac{8n}{100})}{12\,000}$

$\frac{37}{25} = (1 + \frac{8n}{100})$

$\frac{37}{25} - 1 = \frac{8n}{100}$

$100 \times \quad \frac{12}{25} = \frac{8n}{100} \times 100$

$48 = 8n$

$\therefore n = 6$ years

Hire Purchase

- Short term loans (usually to buy household goods) are made using a "hire purchase agreement"
- Simple interest is charged over the given period of the loan.
- After paying the deposit, the balance + insurance + interest is paid off in equal monthly payments (or installments).

Example:

a) James wants to buy a smartphone for R4000. The contract states he must pay a deposit of 10% of the purchase price and interest of 11% p.a. on the balance over 18 months. His monthly insurance premium is R8,50.

Calculate his equal monthly installments.

Deposit: 10% of R4000

$\frac{10}{100} \times 4000 = R400$

Balance owed: R4000 − R400 (cost − deposit)

= R3600

$A = P(1 + in)$

$= 3600(1 + \frac{11}{100} \times 1{,}5)$

$= R\ 4194{,}00$

$A = ?$

$P = 3600$

$i = 11\% \rightarrow \frac{11}{100}$

$n = 1{,}5$ yrs (18 months = 1,5 years)

Installments: (Total owed ÷ Months) + insurance monthly

NB: Monthly!

(R4194 ÷ 18) + R8,50

= R241,50

b) A refrigerator costing R2300 can be bought through hire purchase, with a deposit of R300 and 24 monthly payments of R110 each. Calculate the flat rate of interest charged on the balance.

Balance owed: R2300 − R300
$= R\,2000$

Total loaned: R110 × ~~R~~24
$= R\,2640$

$A = P(1+in)$

A: R2640
P: R2000
i: r% ?
n: 2 (24 months = 2 yrs)

$2640 = 2000\left(1+\frac{r}{100}\times 2\right)$

$\frac{33}{25} = 1 + \frac{2r}{100}$

100 × $\frac{8}{25} = \frac{2r}{100}$ × 100

$32 = 2r$

$\therefore r = 16\%$

Linear Depreciation

- This is similar to simple interest, except instead of there being an increase in money, there is a decrease in money (or worth of an object / asset).
- The formula is the same except instead of a '+' we have a '−'

Formula:

interest rate $\left(\frac{r\%}{100}\right)$

$$A = P(1 - in)$$

n = time periods

Total depreciated amount

initial amount/ worth

Example:

A car worth R40 000 is purchased. Calculate the value of the car after 6 years if linear depreciation occurs at 16% p.a.

$$A = P(1 - in)$$
$$= 40000\left(1 - \frac{16}{100} \times 6\right)$$
$$= R1600$$

A : ?
P : R40000
i : 16% → $\frac{16}{100}$
n : 6

Compound Interest

- Compound interest is the addition of interest to the principal sum of a loan or deposit.
- In other words it is interest on interest, rather than a fixed rate of interest as in simple interest.

Formula:

$$A = P(1+i)^n$$

time periods

interest rate $\left(\frac{r\%}{100}\right)$

Total amount at the end

principal or initial amount

* Note the difference in the formula compared to simple interest.

Examples:

a) If school fees is R5000 in January 2019 for the whole year, at what rate should Harry invest R5000 to get R9000 in January ~~2030~~ 2026 if his bank pays compound interest?

$A = P(1+i)^n$

$9000 = 5000(1+i)^7$

$\sqrt[7]{\frac{9}{5}} = \sqrt[7]{(1+i)^7}$

$1{,}0875957 = 1+i$

$0{,}0875957 = i$

$\therefore i = 8{,}76\%$

A: R9000

P: 5000

i: ?

n: 2026 − 2019 = 7 yrs

b) Nala invests R1500 with her bank over a period of 4 years. If this investment is compounded annually at a rate of 7% interest what will the total of the investment be?

$A = P(1+i)^n$

$= 1500(1+\frac{7}{100})^4$

$= R\ 1966,19$

A: ?
P: R1500
i: 7% → $\frac{7}{100}$
n: 4

c) When Simba was born his mother invested R5000. On his 18th birthday the bank paid him R26 292,89. What was the bank's rate of compound interest per year?

A: R26 292,89
P: R5000
i: ?
n: 18

$A = P(1+i)^n$

$26\ 292,89 = 5000(1+\frac{i}{100})^{18}$

$5,258578 = (1+\frac{i}{100})^{18}$

$1,096599996 = 1+\frac{i}{100}$

$0,096599996 = \frac{i}{100}$

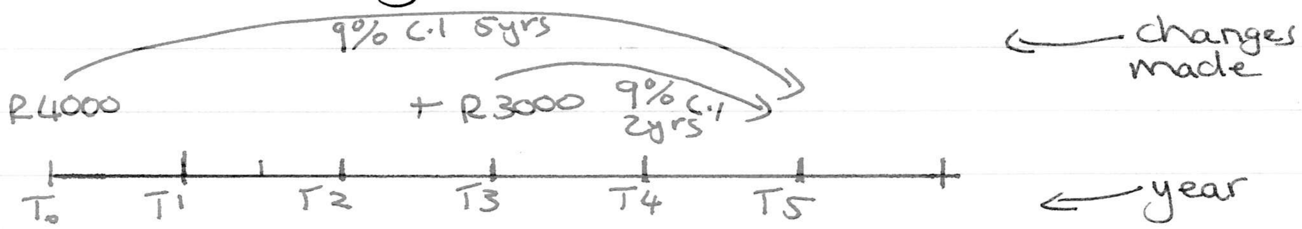
$\therefore 9,66\% = i$

Timelines

- Sometimes interest rates or compound periods change during an investment.
- Investors can also make deposits or withdrawals during their investment period.
- These events affect the final value of an investment.
- The best way to calculate these is by use of timelines which visually represent the information.

Worked example:

R 4000 was deposited in a savings account and 3 years later another R3000 was deposited in the same account. The rate of interest is 9% C.I. Calculate the total amount at the end of 5 years.



$A = P_1(1+i)^{n_1} + P_2(1+i)^{n_2}$

$= 4000(1+\frac{9}{100})^5 + 3000(1+\frac{9}{100})^2$

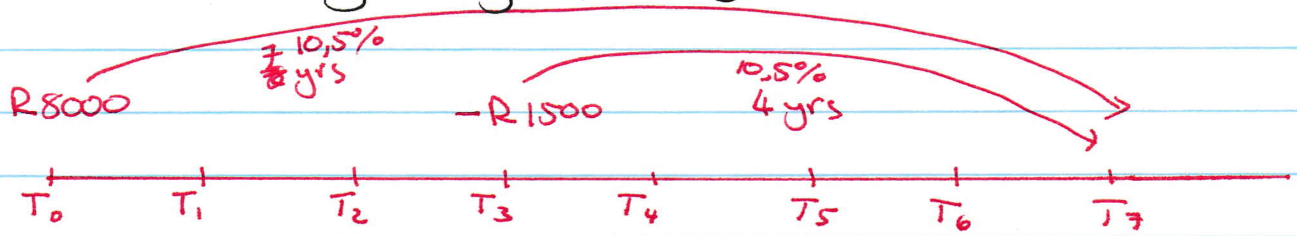
$= R\ 6154{,}49582 + R\ 3564{,}30$

$= R\ 9718{,}80.$

We are combining all calculations into 1. For questions like these we carry each part for the full investment term then add in or subtract the deposits / withdrawals and work out those till the end of term.

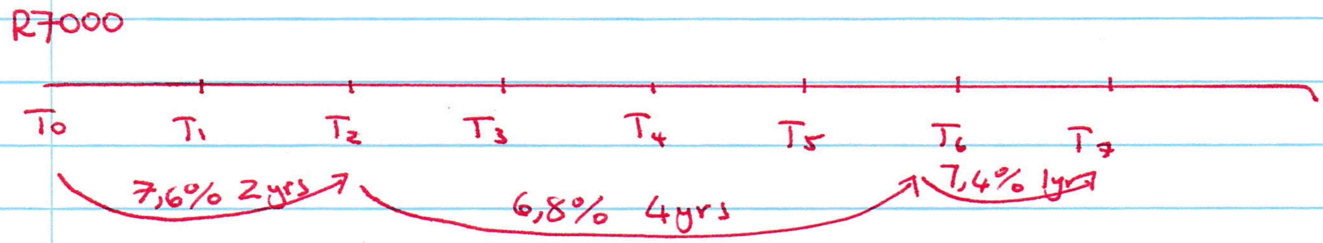
More examples
- Type a : Same rate but withdrawal or deposit
- Type b : Different rates but no withdrawal or deposit
- Type c : Different rates with withdrawals or deposits

a) Simoné deposited R8000 in the Money Market Fund. Three years later she withdrew R1500. How much will be in the account 7 years after the deposit was made if 10,5% p.a. compound interest, compounded yearly is payed?



$$A = P(1+i)^n - P(1+i)^n$$
$$= 8000\left(1+\frac{10,5}{100}\right)^7 - 1500\left(1+\frac{10,5}{100}\right)^4$$
$$= R\ 13\ 856,24$$

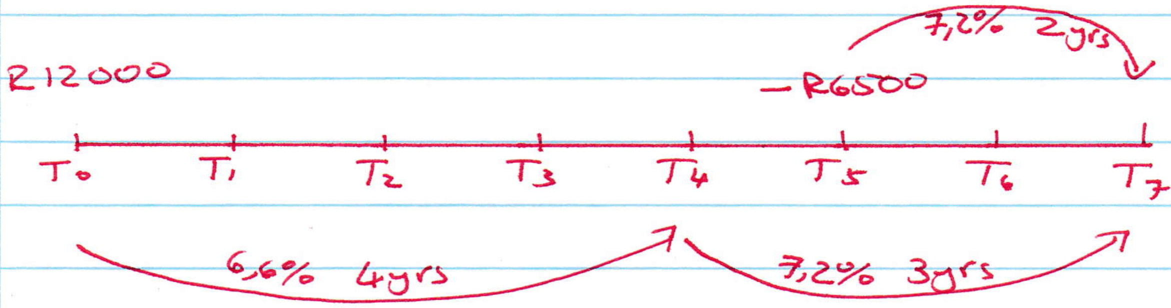
b) Jason invests R7000 in an account for 7 years. The rate of interest is 7,6% CI for 2 years, then 6,8% p.a. CI for 4 years then 7,4% p.a CI for the rest of the period. Calculate the final amount after the 7 years.



$$A = P(1+i)^n(1+i)^n(1+i)^n$$
$$= 7000\left(1+\frac{7,6}{100}\right)^2\left(1+\frac{6,8}{100}\right)^4\left(1+\frac{7,4}{100}\right)^1$$
$$= R\ 11\ 324,31$$

*No deposits or withdrawals hence no + or – between periods.

c) Michael invests R12 000 at 6,6% p.a. CI for 4 years and at 7,2% p.a CI for another 3 years, but withdrew R 6500 after 5 years. Calculate the final amount after the 7 years.



$$A = P(1+i)^n(1+i)^n - P(1+i)^n$$
$$= 12\,000\left(1+\frac{6,6}{100}\right)^4\left(1+\frac{7,2}{100}\right)^3 - 6500\left(1+\frac{7,2}{100}\right)^2$$
$$= R\ 19\,089,494 - R\ 7469,696$$
$$= R\ 11\,619,86$$

Important Theoretical Note

- The compound interest formula can be applied to other areas such as population growth in people, animals and bateria.
- Inflation also follows the same principle as compound interest.