FACTORISING
(PART ONE)

① Common Factor

→ Simplest type

→ We remove the highest common factor of all terms in the expression

Examples:

1) $x^2 + 5x$

$= x(x+5)$ → 'x' is common to both terms in the expression

2) $x^3y^2 - 2x^2y^3$

$= x^2y^2(x-2y)$ → x^2y^2 is common

What we have in brackets is what is required to be multiplied by our common factor in order to get back to the original expression if we were to simplify.

3) $3x(a-b) + 9y(a-b)$

$= (a-b)(3x+9y)$ → common factor is $(a-b)$

4) $k(x-y) - 3(y-x)$ → $(x-y)$ is not the same as $(y-x)$

$= k(x-y) + 3(x-y)$ → Change sign in order to swap signs in 2nd bracket to make common

$= (x-y)(k+3)$

② Difference of squares (DOTS)

→ 2 squares in the expression

→ Need to make 2 brackets rooting each term

→ Different sign in each bracket

Examples:

1) $x^2 - 9$

$= (x-3)(x+3)$ → 2 brackets, different signs; terms square rooted

2) $16x^2 - 25$

$= (4x-5)(4x+5)$

3) $(x^8 - y^8)$ → We may have multiple DOTS in one question

$= (x^4-y^4)(x^4+y^4)$

$= (x^2-y^2)(x^2+y^2)(x^4+y^4)$

$= (x-y)(x+y)(x^2+y^2)(x^4+y^4)$

4) $(x+2y)^2 - 9$ → DOTS with brackets

$= (x+2y-3)(x+2y+3)$

↳ Make sure to only change the signs of the sign of that separates the 2 squares

③ Trinomials (part 1)

→ These have 3 terms ('tri') with a square (eg: $x^2+13x+12$)

→ These can be done with a calculator, or manually

→ We want to get 2 brackets

Examples

1) $x^2+7x+10$

$= (x+5)(x+2)$

10	5
+ 1	+ 2
11	7

Write out factors of the constant (10) → they need to add to 7.

$x^2 + 7x + 10$

1) tells us if the signs are the same (+ = same, − = different)

2) tells us what the sign is if they're the same, or if they are different the bigger number gets this sign

2) $x^2 - 2x - 3$ → signs are different

$= (x-3)(x+1)$

bigger number gets the '−'

FACTORISING
(PART TWO)

3 Trinomials (part 2)

1) $2x^2 - 13x - 15$

$= (2x-15)(x+1)$

$2x \; \cancel{-3}\,15$
$x \; + \cancel{5}\,1$

$-15x$
$2x$
$-13x$

→ Write out factors of 1st term and 3rd term.

→ We need to get the right combo to reach the 2nd term

2) $6x^2 + 5x - 4$

$= (3x+4)(2x-1)$

$3x + 4$
$2x - 1$

$+8x$
$-3x$
$5x$

5 Cubes (difference or addition)

1) $x^3 - 1$

$= (x-1)(x^2+x+1)$

1. Make a small bracket + big bracket
$(\quad)(\quad\quad)$

2. Small bracket: Cube root both terms keeping the sign
$x^3 - 1$
$(x-1)(\quad)$

3. Big bracket: Using the new small bracket:
   1) Square 1st term $x \rightarrow x^2$
   2) Multiply the terms together & change the sign $(x)\times(-1) = -x \rightarrow +x$
   3) Square the 2nd term → keep sign as positive $(-1)^2 \rightarrow 1$

$(x-1)(x^2+x+1)$

2) $27x^3+1$

$= (3x+1)(9x^2-3x+1)$

NB:
For factorising questions can combine several different types of factorising discussed. Pay attention and think carefully!

4 Grouping

→ Usually 4 terms where we need to group them into 2 groups of 2 based on common factors

→ Your answer is likely to form 2 brackets.

Examples:

1) $2x+2y+kx+ky$ → Group

$= 2(x+y) + k(x+y)$ → take out a common factor from each pair

$= (x+y)(2+k)$ → Common factor as in 1

2) $x^2 - xy + xz - zy$

$= x(x-y) + z(x-y)$

$= (x-y)(x+z)$

3) $6x^2 + 3xy - 2xt - yt$

$= 6x^2 - 2xt + 3xy - yt$

$= 2x(3x-t) + y(3x-t)$

$= (3x-t)(2x+y)$

4) $x^2 - 9y^2 - x - 3y$

$= (x-3y)(x+3y) - 1(x+3y)$

$= (x+3y)[(x-3y)-1]$

$= (x+3y)(x-3y-1)$